ICSV13 - Vienna
The Thirteenth International Congress on Sound and Vibration
Vienna, Austria, July 2-6, 2006

VIBRATION TRANSMISSION ACROSS PLATE JUNCTIONS – A BRIEF REVIEW

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Abstract

Despite the fact that the phenomenon of wave transmission in thick connected plates has been studied over many years, this paper is aimed at doing a brief review of the problem. It is part of a greater research project concerning the prediction of traffic noise at bus terminals in Uberlândia City, Brazil. At this initial stage, the main task is to predict the vibration transmission at junctions of connected thick slabs and walls, typically used in conventional buildings in Brazil. In addition, an assessment of the influence of the joint configuration was also considered. Thus, the contribution of this analytical study was to provide some information concerning transmission efficiency of waves propagating along typical building elements at low, mid and high frequency ranges; and subsequently to identify the governing parameters that affect the overall sound pressure level inside the bus terminals. Moreover, the influence of joint layouts on transmission was to be examined.

INTRODUCTION

Although structure-borne noise transmission at structural junctions has been investigated over many years [1,2], increased efforts have recently been directed to satisfy minimum sound insulation criteria recommended by stricter regulations. In building acoustics many studies of vibration transmission at most common joints have been based on simplified models, which are generally described in terms of line connected ‘thin’ plates. Even though other types of waves propagate in flat thin plates, e.g. those known as in-plane waves, flexural waves are generally the most important ones when noise radiation from floors, ceilings and/or walls in buildings is being considered. The thin plate theory is primarily based on the assumption that the bending wavelength of the propagating bending wave, in a thin plate, must be sufficiently larger than the plate thickness. A practical limit of validity for this theory has been established [3]. On the other hand, some types of sources (such as footsteps, dropping of an object on a hard surface floor, etc.) may lead to high-frequency energy transmission in a building structure. Thus, the 'thin' plate theory, which is described in the next section, may not be appropriate in all of these circumstances. As a result, a more complex formulation, 'thick' plate theory, was developed to overcome the limitations imposed by the simplified theory.

Despite the fact that the 'thick' plate theory has already been used for various problems in building structures [6], the main concern initially was to achieve an optimum balance between lightweight components and stability of the whole 'skeleton'. The consideration of the connection of thin and thick building elements (such as floors, ceilings and walls) and its effects in terms of structure-borne sound transmission has already been studied in previous publications [4,5,7,8]. There are some limitations in the reported studies and further study is still required for the development of a more general model. In addition, the analysis of the sensitivity in the results, in terms of coupling loss factors, has not yet been fully explored in terms of considering different joint configurations.

The results are presented for the most common type of structural junction namely the corner joint. Finally, the main conclusions are presented.

THE WAVE DYNAMIC STIFFNESS METHOD (WDS)

The WDS method was developed previously for thin plates [9]. In this method a dynamic stiffness matrix related displacements to forces acting on a structure, and could be derived for finite and infinite plates. It was assumed that plate boundary is infinite in length along the connection. Hence, a 'wave' dynamic stiffness matrix describes displacements along an infinite panel boundary to the forces applied to the boundary. The wave solutions were related to the boundary displacements on the semi-infinite panel by the matrix equation [9]

$$\mathbf{B}_j \boldsymbol{\zeta}_j = \mathbf{d}_j \tag{1}$$

where $\mathbf{B}_j$ is a square matrix obtained by using the solution to the wave equations [9], $\boldsymbol{\zeta}_j$ is a column vector containing the amplitudes of the wave solutions considered on a particular plate and $\mathbf{d}_j$ is a column vector containing the displacements along the boundary. The subscript *j* indicates semi-infinite plate *j*. Similarly, the relationships between panel deflections and boundary forces are defined as

$$\mathbf{C}_j \boldsymbol{\zeta}_j = \mathbf{f}_j \tag{2}$$

where $\mathbf{C}_j$ can be obtained by using the strain-displacement and stress-strain relationships [9]. Multiplying equation (1) by $\mathbf{B}_j^{-1}$ and substituting $\boldsymbol{\zeta}_j$ into equation

(2)

$$\mathbf{C}_j\,\mathbf{B}_j^{-1}\mathbf{d}_j = \mathbf{f}_j \tag{3}$$

Thus, the stiffness matrix of a semi-infinite panel, in which the flexural and extensional vibrations are included, is defined as

$$\mathbf{K}_{L\,j} = \mathbf{C}_j\,\mathbf{B}_j^{-1} = \begin{bmatrix} \mathbf{K}_f & 0 \\ 0 & \mathbf{K}_e \end{bmatrix}_j \tag{4}$$

where $\mathbf{K}_f$ and $\mathbf{K}_e$ are the flexural and extensional stiffness matrix contributions [9].

The global stiffness matrix is calculated as

$$\mathbf{K}_{Gj} = \mathbf{R}_j\,\mathbf{K}_{Lj}\,\mathbf{R}_j^{-1} \tag{5}$$

where $\mathbf{R}_j$ is the transformation matrix for one (semi-infinite panel) or two (strip-plate) 'nodes' elements. It relates displacement vector $\mathbf{d}_{Gj}$ in global coordinates to the local displacement vector $\mathbf{d}_{Lj}$ as

$$\mathbf{d}_{Lj} = \mathbf{R}_j^{-1}\mathbf{d}_{Gj} \tag{6}$$

Subsequently, the joint stiffness matrix $\mathbf{K}_{GT}$ is determined using a matrix assembly method [10]. For instance, the joint stiffness for semi-infinite panels connected at a 'line node' is simply given by

$$\mathbf{K}_{GT} = \mathbf{K}_{Ginc} + \sum_{j=1}^{N}\mathbf{K}_{Gj} \tag{7}$$

where $\mathbf{K}_{Ginc}$ is dynamic stiffness of the panel ('input panel') which holds the incident wave [9], $\mathbf{K}_{Gj}$ is the dynamic stiffness of the remaining panels [9] which share the same connection, and N refers to the total number of connected panels to the 'input panel'. On the other hand, if an infinite 'beam element' is used [11] rather than an infinite line at the junction, its stiffness S_b must also be included into equation (7). In order to determine the displacement at the common junction, the force $\mathbf{f}_{GT}$ applied to the joint remains to be defined. This vector may be expressed as [9]

$$\mathbf{f}_{GT} = -\sum_{j=1}^{N}\mathbf{K}_{Gj}\mathbf{d}_{INP} - \mathbf{f}_{INP} \tag{8}$$

where $\mathbf{d}_{INP}$ is the global deflection of the joint [9], which for instance may have more than one node (e.g. 'strip plate' included), due to an incident excitation wave. In addition, $\mathbf{f}_{INP}$ is the force vector acting on the 'nodes' of the joint due to the presence of the incident wave. Finally the system is then solved in terms of the wave amplitudes and resulting displacements as

$$\mathbf{K}_{GT}\ \mathbf{d}_p = \mathbf{f}_{GT} \tag{9}$$

where $\mathbf{d}_p$ is the displacement of the joint due to the force $\mathbf{f}_{GT}$. Thus, the total displacement of the joint is given by

$$\mathbf{d}_{GT} = \mathbf{d}_{INP} + \mathbf{d}_p \tag{10}$$

The complex amplitudes of the traveling waves in each panel may be calculated by relating the local displacements to the wave amplitudes (Equation 1). For instance, the wave amplitudes on semi-infinite connected plates connected on a single node are

$$\mathbf{w}_{INP} = \mathbf{B}_n^{-1}\mathbf{R}_n^{-1}\mathbf{d}_p \tag{11}$$

$$\mathbf{w}_j = \mathbf{B}_j^{-1}\mathbf{R}_j^{-1}\mathbf{d}_{GT} \tag{12}$$

where $\mathbf{w}_{INP}$ and $\mathbf{w}_j$ are the wave amplitudes of the 'input plate' and 'receiving plates' respectively [9].

THE TRANSMISSION AND REFLECTION EFFICIENCIES

The transmission and reflection efficiencies were defined as the ratio of power flow per unit length from the boundary to the power per unit length incident upon the common boundary. In other words, the incident, reflected and transmitted intensities normal to the junction were to be determined. The intensities were evaluated for thick plates. It was assumed that the frequency-range considered in this work was much lower than the frequencies corresponding to the first modes of the thickness-shear and torsional motions. Therefore, the Mindlin's bending phase speed is used to evaluate the flexural propagating power flow [13]. The in-plane motion was assumed to be decoupled from the thickness vibration modes. Thus, the calculations were only considered at frequencies which are lower than the frequency of the first thickness-stretch mode of vibration for the plate. It is well-known that the sum of transmission and reflection efficiencies must be unity in order to ensure the conservation of energy in a joint [3]. In this paper, an incident propagating Mindlin bending wave was considered. Furthermore, the transmission and reflection efficiencies were averaged over all incident angles. Thus, a diffuse sound field was assumed in the incident plate.

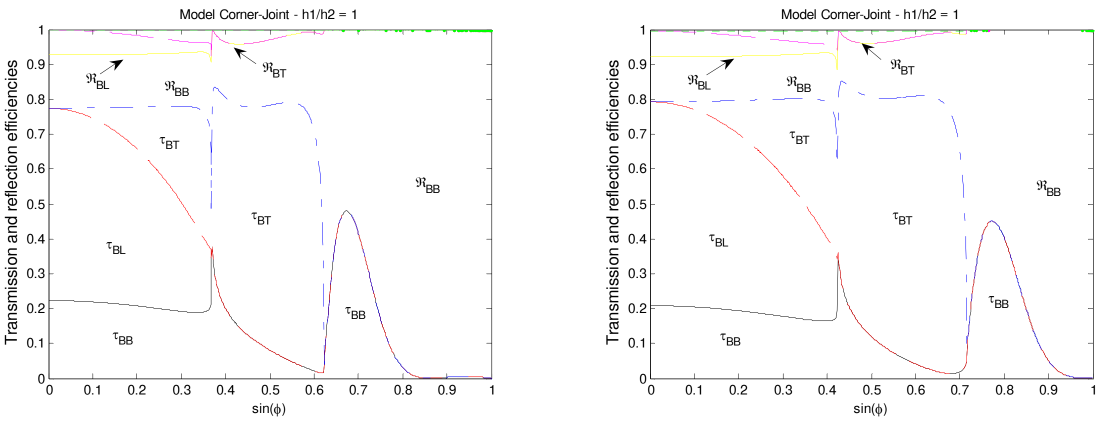
NUMERICAL RESULTS

The results presented here take into account rotational inertia and shear deformation effects in the calculations. Also in these simulations the in-plane waves generated at the junctions are fully incorporated in the solutions. It is known that the effectiveness of connections on sound transmission does not only depend on material properties but also on the geometry of joint configurations. The calculations were obtained for two types of joints. For the corner-type joint 1, concrete plates were considered for both legs. For the corner-type joint 2 the material properties of concrete and masonry were considered for the source and receiving legs respectively. The assumed values for the Young's modulus, Poisson's ratio and density of concrete were 15 GN/m^2, 0.3 and 2400 kg/m^3 respectively. Likewise, the properties for the masonry were 5 GN/m^2, 0.2 and 1800 kg/m^3. The geometrical properties of the plates also contribute to the effectiveness of subsystem to subsystem transmission. A thickness value of 0.1 m was assumed for all of the semi-infinite plates. The line-connection 'L' configuration considered here corresponds to one of the benchmark models proposed recently in ref. [15]. Nevertheless, the effects of rotational inertia and shear deformation were considered in the WDS method for a corner joint-description model. There exist 5 degrees of freedom along the boundary. The interaction of the incoming, transmitted and reflected waves satisfy the boundary conditions. The governing equations are presented in ref. [13]. Five types of distinct waves can exist in a thick plate. They are the pure bending wave, the bending near field, quasi-longitudinal waves and transverse waves.

The incident intensity considered in the model was due to a propagating flexural wave in the semi-infinite plate 1, which is made of concrete. As mentioned before, the coupling between distinct types of propagating waves (e.g. Mindlin-bending and 'in-plane' waves) was also considered. For the sake of simplicity, the term 'longitudinal waves' was referred to 'quasi-longitudinal' waves herein. Figures 1 and 2 show the variation with angle of incidence of transmission and reflection efficiencies, taking into account not only secondary longitudinal and transverse waves but also Mindlin bending waves. It is well-known that no energy (including secondary longitudinal and transverse wave intensities) can be transported beyond the boundary line at certain limiting angles of incidence. Distinct limiting angles of incidence can be obtained for the reflected and transmitted waves. At frequencies for which the bending phase speed in the receiver is much lower than either the quasi-longitudinal or transverse shear wave speeds, the transmitted in-plane waves do not contribute to the calculated power flow for most angles of incidence. The efficiencies sum up to unity, as it is required to ensure the conservation of energy for a non-dissipative junction transmitted waves. The transmission efficiencies for both corner-joint types 1 and 2 were calculated for a frequency range 0-2500 Hz and shown in Figures 3-(a) and 3-(b) respectively. Angular average values of transmission coefficients were obtained. The integration was performed numerically using

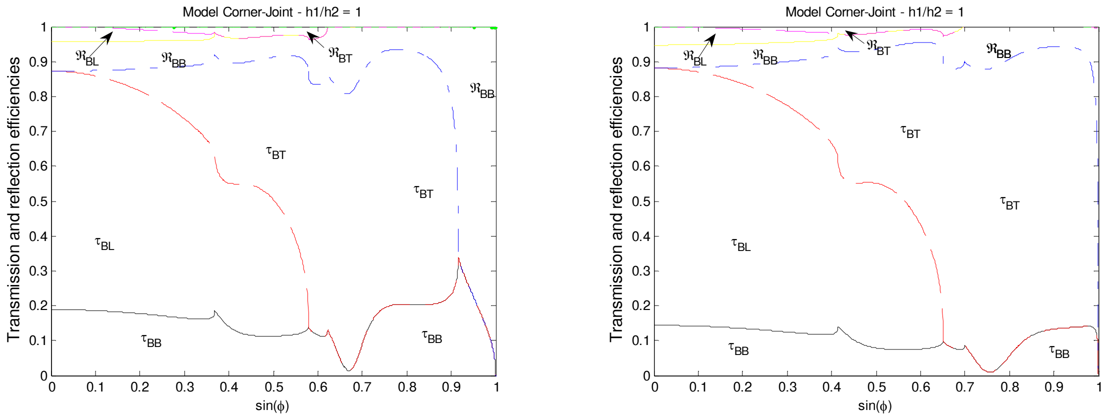
Simpson's Rule and assuming diffuse field incidence. The length of time required for the calculation was directly proportional to the number of angles computed (= 1,000) and the highest frequency considered (= 2500 Hz). For the corner-type joint 1, which is composed of identical plates, a maximum of 50 % of the normal incident energy can be transmitted into the receiving plate in terms of a propagating bending wave [3]. The results show the TL values obtained from the combined Mindlin bending and in-plane formulation which is compared with the values for the conditions of thin-plate bending, Mindlin bending and the thin-plate bending plus in-plane analysis. The results show the corresponding angular averaged TL for an incident out-of-plane bending wave for the **L** configuration.

Figure 3-(a) shows that for the thin-plate theory and the Mindlin bending wave formulations, the transmission loss (TL) values differ slightly from each other. Furthermore, it is seen that the decrease of TL was significant for values of $kh > 1$ ($\approx 1100\,\text{Hz}$), where h is the plate thickness and k is the trace wavenumber corresponding to flexural motion. Figure 3-(b) shows that the decrease of TL was significant at frequencies greater than 600 Hz. In the Statistical Energy Analysis (SEA) method, the power flow between coupled structural plates (components) is determined by considering a power balance for each component. The coupling loss factor (CLF) describes the nature of coupling between the plates. For a line junction, the calculation of the CLF is based on the assumption that the transmission coefficient is much less than one. Nevertheless, this might not be true at frequencies for which in-plane waves are significant.



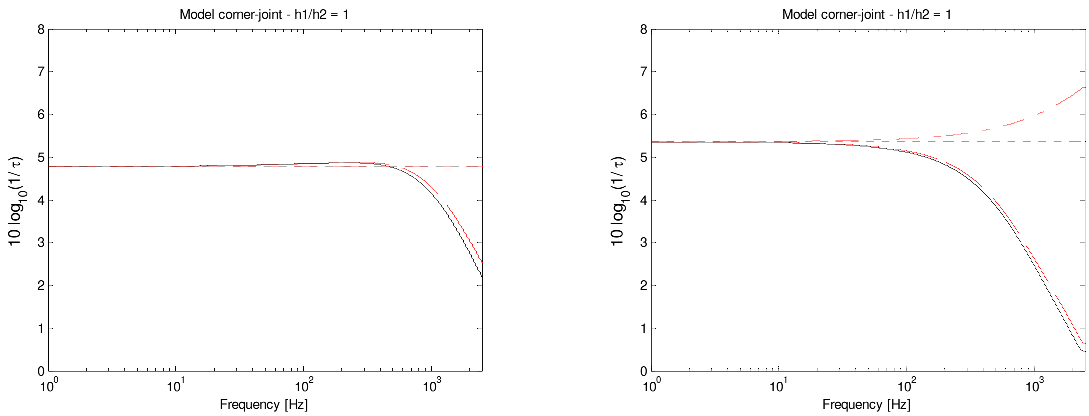
Thin-Plate Bending (2000 Hz) a) *Mindlin-Plate Bending (2000 Hz)* b)

Figure 1: *Transmission* (τ) *and Reflection* $(\Re)$ *efficiencies for the corner-type joint 1 at 2000 Hz. a) Thin-plate bending; b) Mindlin bending; The subscripts shown in the figure represent: BB – Bending to Bending; BL – Bending to Longitudinal; BT – Bending to Transverse.*



Thin-Plate Bending (2000 Hz) a) *Mindlin-Plate Bending (2000 Hz)* b)

Figure 2: *Transmission* (τ) *and Reflection* $(\Re)$ *efficiencies for the corner-type joint 2 at 2000 Hz. a) Thin-plate bending; b) Mindlin bending; The subscripts shown in the figure represent: BB Bending to Bending; BL – Bending to Longitudinal; BT – Bending to Transverse.*



a) b)

Figure 3: *Angular averaged transmission loss (TL) for:a) the corner-type joint 1; b) the corner-type joint 2 ..... Bending (thin plate); _____ Bending + in-plane waves; -.-.-.- Bending (Mindlin's thick plate – red colour); ------ Bending (Mindlin's thick plate)+ in-plane waves – red colour*

CONCLUSIONS

It can be observed that the main effect of including in-plane waves in the analyses was to increase the transmitted intensity at the junction. In other words, the effects of in-plane waves on the transmission efficiencies show that for a corner junction, the predictions might lead to significant underestimation of the values, especially for increasing frequency considerations. In addition, the effects of shear deformation and rotary inertia were significant at higher frequencies. Although a formal benchmarking of numerical models for computing transmission coefficients [15] was set to facilitate the transfer of information between SEA users, the TL values were computed only in terms of semi-infinite thin-plate junctions. Therefore, the inclusion of the thick plate bending effects might give some additional insight to existing wave transmission problems in SEA analysis.

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